Theorem:- The necessary and sufficient conditions for a curve to be helix is that its curvature and torsion are in constant ratio.

OR

Curve is helix $\Leftrightarrow \frac{K}{\tau} = \text{constant}$

Proof: Necessary Condition – Let curve to be helix, then show $\frac{K}{\tau} = \text{constant}$

Since curve is helix then generator $(\vec{a})$ cuts the cost. angle with curve

i.e $\vec{t}\cdot\vec{a} = \cos\alpha = \text{const}$ —— (1)

But α is the const. angle between unit generator $(\vec{a})$ and unit tangent $(\vec{t})$

Now diff. (1) w.r.t 's'

$$\frac{d}{ds}(\vec{a}\cdot\vec{t}) = 0$$

$$\Rightarrow \frac{d\vec{a}}{ds}\cdot\vec{t} + \vec{a}\cdot\frac{dt}{ds} = 0$$

$$\Rightarrow 0 + \vec{a}\cdot\frac{d\vec{t}}{ds} = 0$$

$$\Rightarrow \vec{a}\cdot\vec{t}' = 0$$

but $t' = Kn$

$a\cdot Kn = 0$

$\Rightarrow K(a\cdot n) = 0$

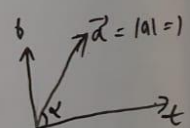
$\Rightarrow$ either $K=0$ or $a\cdot n=0$

but $K \neq 0$ otherwise curve will be straight line

$\Rightarrow a\cdot n = 0$

$\Rightarrow a \perp$ to n

but rectifying plane is $\perp n$



$\Rightarrow$ a lies in rectifying plane

i.e a contain t and b

then $a = \cos\alpha\cdot t + \sin\alpha\cdot b$

diff w.r.t 's' we get

$0 = \cos\alpha\cdot t' + \sin\alpha\cdot b'$

$0 = K\cos\alpha\cdot n - \tau\sin\alpha\cdot n$

$0 = n\cdot\{K\cos\alpha - \tau\sin\alpha\}$

$\because t' = Kn$, $b' = -\tau n$

but $n \neq 0$ as unit principal normal

$K\cos\alpha - \tau\sin\alpha = 0$

$\Rightarrow K\cos\alpha = \tau\sin\alpha$

$\Rightarrow \frac{K}{\tau} = \frac{\sin\alpha}{\cos\alpha} = \tan\alpha = \text{Constant}.$